

HYDRODYNAMIC AND THERMAL CHARACTERISTICS
OF AN EQUILIBRIUM TWO-PHASE
POROUS-COOLING SYSTEM

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Static characteristics are constructed for determining the permissible fluctuations in the governing parameters and for determining the features of stable two-phase porous cooling.

Increases in the operating capacity of various types of equipment impose stiffened requirements on reliable heat-shielding systems, among which the most promising is that in which a liquid undergoes a phase transition within a heated porous surface. The fact that no practical application of this method has yet been reported reflects a serious, as yet unsurmounted obstacle: the instability of two-phase porous cooling.

One of the basic reasons for this instability was established in [1] through analysis of the solution of the nonlinear closed system of differential equations describing the physical model for two-phase porous cooling. Below we continue the analysis of this process by constructing and analyzing its static characteristics, which are analogous to the characteristics of other heat-engineering systems whose operation is known to be unstable.

A system (in particular, the two-phase porous-cooling system with which we are concerned here) is aperiodically unstable if, after a deviation from the steady state resulting from a small fluctuation, there is no new steady state near the original one, and the state parameters instead undergo a large-amplitude monotonic change. The reason for this behavior lies in the laws of the steady state, which are described by equations in which there are no time derivatives. The static characteristics are a reflection of these laws, and analysis of these characteristics can not only show whether the system is aperiodically unstable but also reveal the permissible slow changes in parameters, i. e., those for which the process remains stable.

Static Characteristics of Unstable Heat-Engineering Systems. Two types of aperiodic instabilities of heat-engineering equipment are known at present: the "boiling crisis" and the sudden, large change in the flow rate of the working medium in a heated channel. For the boiling of a liquid in a large volume the static characteristic is the boiling curve, i. e., the heat flux from the heating surface to the liquid as a function of the temperature drop. Typical boiling curves, for nitrogen and Freon-113, are shown in Fig. 1. Figure 2 shows an illustrative hydrodynamic characteristic of a heated channel, which gives the channel resistance as a function of the flow rate of the working medium. A distinctive feature of the static characteristics for both processes is that they are not single-valued: corresponding to a single value of the independent parameter (along the ordinate) are three quite different values of the parameter plotted along the abscissa (points B, D, and F). The descending part (CE) of the boiling curve reflects the decrease in the heat flux with an increase in the temperature drop at the transition from nucleate boiling to film boiling, while region CE on the hydrodynamic characteristic corresponds to the decrease in the channel resistance resulting from a decrease in the vapor content of the two-phase flow.

The cooling of a solid wall by a boiling liquid remains stable until the slopes of the internal and external characteristics of the system satisfy the following inequality at the intersection of these characteristics [5, 6]:

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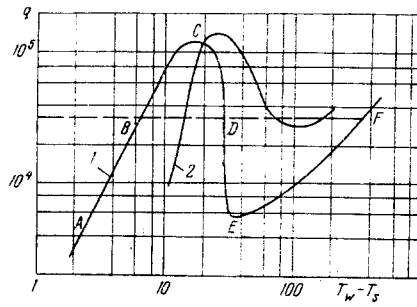


Fig. 1

Fig. 1. Characteristic boiling curves for a liquid in a large volume. 1) Nitrogen [2]; 2) Freon-113 [3]. Here q is in kilocalories per square meter per hour and $T_w - T_s$ is in Celsius degrees.

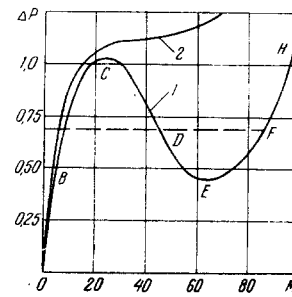


Fig. 2

Fig. 2. Hydrodynamic characteristics for a uniformly heated channel [4]. 1) Multivalued; 2) single-valued. Here ΔP is in bars and M is in kilograms per hour.

$$\frac{dq_{\text{ext}}}{dT_w} < \frac{dq_{\text{int}}}{dT_w} \quad (1)$$

The boiling curve is an internal characteristic of the system reflecting its particular properties, while the external characteristics are characterized by the relationship between the heat flux q_{ext} supplied by external sources and the wall temperature T_w .

It is common to find processes (electrical heating, radiative heating, and heat evolution in fuel cells) in which the heat flux to the surface is nearly or completely independent of the temperature, $dq_{\text{ext}}/dT_w = 0$. In such situations there can be a transitional boiling regime with $dq_{\text{int}}/dT_w < 0$ because the stability condition is violated. The critical heat flux, corresponding to point C, is the boundary for stable and reliable operation.

A steady state can be achieved in the transitional boiling regime only if rapid external heat transfer can be achieved in some manner (e.g., by means of vapor condensation), i.e., only if the heat-transfer coefficient h_{ext} for this process is larger than the local slope of the descending part of the boiling curve: $h_{\text{ext}} = -dq_{\text{ext}}/dT_w = -d/dT_w[h_{\text{ext}}(T_\infty - T_w)] > -dq_{\text{int}}/dT_w$. However, because of the limited value of the heat-transfer coefficient and the important destabilizing effect of the thermal resistance of the wall, a steady state can actually be achieved in this manner only at the beginning and end of the transition regime, where the slope of the boiling curve is small [7, 8]. Accordingly, to obtain data over the entire transition regime one makes use of either additional cooling of the heat-evolving convection element (in addition to the cooling by boiling of the main heat-transfer surface in the steady state [3, 9]) or an unsteady regime of cooling by boiling of the initially hot medium [2, 3].

The motion of a liquid in a heated channel is stable if the slope of the hydrodynamic (internal) characteristic of the channel at the working point is algebraically larger than the slope of its external characteristic (the curve showing the total flow rate M against the pressure drop ΔP_{ext} over the channel, produced by a pump) [10]:

$$\frac{d\Delta P_{\text{ext}}}{dM} < \frac{d\Delta P_{\text{int}}}{dM} \quad (2)$$

If a constant pressure drop $d\Delta P_{\text{ext}}/dM = 0$ is maintained over the heated channel, the descending part of the channel characteristic, $d\Delta P_{\text{int}}/dM < 0$, is a region of unstable operation. Under these conditions the aperiodic instability can be avoided by making the hydrodynamic characteristic single-valued; this is done by inserting an additional resistance — a constricting washer — at the entrance to the channel, where a single-phase liquid is flowing. Together with the pressure drop in the channel, the resistance of this washer stabilizes the process. Curve 2 in Fig. 2 shows the hydrodynamic characteristic of the stable system.

It is important to note that the descending region on the hydrodynamic characteristic is not peculiar to systems in which the working medium undergoes a phase transition; it turns out that the hydrodynamic characteristics of a gas-cooled porous plate has a shape like that of curve 1 in Fig. 2, for both volume [11] and surface [12] heat exchange. Here the multivalued nature of the characteristic is a consequence of the temperature dependence of the viscosity of the gaseous coolant and has the consequence that three mass flow rates of the coolant (with quite different average wall temperatures) correspond to a certain pressure drop at the plate. The region of stable and reliable operation is branch EH, where the coolant flow can be controlled effectively by changing the pressure drop at the plate and where the flow rate is sufficient to maintain the temperature of the material within a permissible range. A decrease of the pressure drop below the value corresponding to point E leads to a sharp decrease in the coolant flow rate, which is accompanied by wall burnout. In contrast with systems in which the working medium undergoes a phase transition, there is no way to change the hydrodynamic characteristic of the gas-cooled porous plate from multivalued to single-valued.

It follows that the static characteristics can be used to determine the permissible quasisteady fluctuations in the parameters (in the density of the incoming heat flux in the case of boiling or in the pressure drop along the plate in the case of gaseous porous cooling) for which the system continues to operate stably and reliably. As for two-phase porous cooling, we note that the static characteristic constructed in [1] (the dependence of the pressure on the temperature in the presumed phase-transition region), the most natural result of a solution of the equations describing the process, reveal in a straightforward manner only the permissible fluctuations in the initial coolant temperature. Because of the particular features of the operation of this system it is equally interesting to consider the determination of the permissible fluctuations in other governing parameters, e.g., the external heat flux and the pressure drop at the plate. Such a problem can be solved by means of characteristics including the parameter whose permissible fluctuations are to be determined.

Hydrodynamic Characteristic of an Equilibrium Two-Phase Porous-Cooling System. We adopt the physical model and the notation of [1] without any changes. Below we will also need some quantitative relations derived in [1], to which we turn now. Under the assumption that the coolant undergoes a phase transition within a porous plate, the specific flow rate of the coolant, G , is related to the pressure drop $P_0 - P_1$ at the plate by

$$P_0 - P_1 = \delta \left\{ \alpha G [v'l + v''(1-l)] + \beta G^2 \left[\frac{l}{\rho'} + \frac{(1-l)}{\rho''} \right] \right\}, \quad (3)$$

where δ is the plate thickness; l is the dimensionless coordinate of the phase-transition surface; α and β are the viscous and inertial drag of the porous structure; and ρ' , ν' , ρ'' , ν'' are the density and kinematic viscosity of the liquid and vapor, calculated for the saturation state at a pressure equal to the given pressure of the external medium, P_1 .

The drag of the vapor part of the coolant motion is a part of the drag of the entire plate:

$$P_l - P_1 = \delta \left[\alpha G v'' (1-l) + \beta \frac{G^2}{\rho''} (1-l) \right], \quad (4)$$

where P_l is the pressure in the phase-transition region.

The temperature in this region is calculated from

$$(T_l - T_\infty) \frac{c'}{r} = \frac{q \exp \left[\frac{G \delta c''}{\lambda_2} (l-1) \right]}{Gr} - 1, \quad (5)$$

where T_∞ is the initial temperature of the liquid coolant and q is the external heat flux.

The hydrodynamic characteristic of the two-phase porous-cooling system gives the total pressure drop at the wall as a function of the specific flow rate. It would seem at first glance that Eq. (3) alone would be sufficient to satisfy this requirement, but this is not the case: Equation (3) holds only under the assumption that the coolant undergoes a phase transition in the region with coordinate l , regardless of whether a transition can actually occur here. For fixed parameters of the system there is a certain relationship between the specific coolant flow rate and the position of the phase-transition surface for which the pressure and temperature in the evaporation zone correspond to the saturation state. In this case we can say that a phase transition is "probable." A "probable" phase transition becomes "possible" if the system is stable in this state, and, finally, the phase transition becomes "real" if the temperature of the external wall surface does not exceed the limiting temperature.

Accordingly, the condition for the thermodynamic equilibrium of the phase transition is fundamental for constructing a hydrodynamic characteristic for use in evaluating the stability of the system at some working point. We will use the expression "equilibrium model" for this model of two-phase porous cooling with a phase transition of the coolant which is at thermodynamic equilibrium and with a local thermal equilibrium between the wall material and the coolant, in both flow regions.

As one of the parameters governing the equilibrium nature of the phase transition in the region with coordinate l it is convenient to use the enthalpy i_l'' of the dry saturated vapor which is formed instead of its temperature T_l , since Eq. (5) then reduces to the simpler, more accurate form

$$i_l'' - c_m T_\infty = \frac{q \exp \left[\frac{G \delta c''}{\lambda_2} (l - 1) \right]}{G}. \quad (6)$$

We transform this equation by means of

$$i_l'' - i_{l=1}'' = \frac{q \exp \left[\frac{G \delta c''}{\lambda_2} (l - 1) \right]}{G} - (i_{l=1}'' - c_m T_\infty) \quad (7)$$

so that we can determine both the enthalpy of the saturated vapor and the pressure in the phase-transition region from the known saturation parameters for evaporation on the outer surface of the wall, where the pressure P_1 of the surrounding medium is specified. The pressure difference and the enthalpy of the dry saturated vapor are related, in the case of equilibrium phase transitions within the plate and on its external surface, by some one-to-one correspondence

$$i_l'' - i_{l=1}'' = \psi(P_l - P_1), \quad (8)$$

which is governed solely by the nature of the coolant.

Using the linear approximation of this dependence at the point where the pressure is equal to the pressure of the surrounding medium,

$$i_l'' - i_{l=1}'' = \left. \frac{di''}{dP} \right|_{P=P_1} (P_l - P_1), \quad (9)$$

we can combine (4) and (7) into a single transcendental analytic equation for the coolant flow rate G for the case of an equilibrium phase transition in region i :

$$\begin{aligned} & \frac{q \exp \left[\frac{G \delta c''}{\lambda_2} (l - 1) \right]}{G} - (i_{l=1}'' - c_m T_\infty) = \\ & = \left. \frac{di''}{dP} \right|_{P_1} \delta \left[\alpha G v'' (1 - l) + \beta \frac{G^2}{\rho''} (1 - l) \right]. \end{aligned} \quad (10)$$

The necessary total pressure drop at the plate is calculated from the known flow rate by means of Eq. (3). A systematic solution of Eqs. (10) and (3) for all values of the parameter l , followed by the elimination of this parameter, also gives the form of the hydrodynamic characteristic for a two-phase porous-cooling system. The parameters for this process are the external heat load, the initial coolant temperature, the nature of the coolant, the pressure of the surrounding medium, and the physical characteristics of the porous plate.

The solution of implicit Eq. (8) or even of its simplified analytic version (10) requires laborious calculations, but there is one particular case in which the calculations are not as formidable. Over the broad pressure range from 1 to 120 bars the enthalpy of saturated water vapor remains constant within 4.5%: $i'' = \text{const}$ [13]. In this case we can use, instead of (10), the following simpler characteristic equation to determine the specific flow rate in the water-cooled system:

$$\frac{q \exp \left[\frac{G \delta c''}{\lambda_2} (l - 1) \right]}{G} = (i'' - c_m T_\infty). \quad (11)$$

Figure 3 illustrates the solution of Eqs. (11) and (3); for ease in comparison with the data of [1] we have used the same parameters for curves 1-3: the coolant is water, initially at $T_\infty = 20^\circ\text{C}$; the pressure of the surrounding medium is $P_1 = 10$ bars; the wall thickness is $\delta = 5$ mm; the porosity is $\Pi = 0.2$; and the drag coefficients are $\alpha = 3.5 \cdot 10^{13} \text{ m}^{-2}$ and $\beta = 1.2 \cdot 10^8 \text{ m}^{-1}$. The heat flux and the effective thermal conductivity of the vapor region are the same.

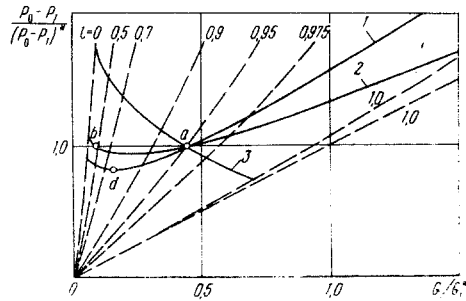


Fig. 3

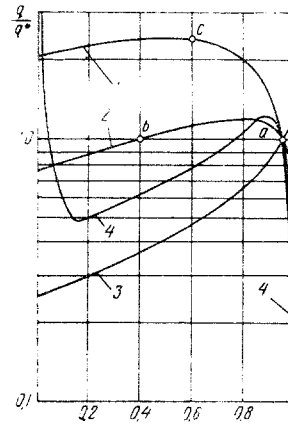


Fig. 4

Fig. 3. Hydrodynamic characteristic of an equilibrium two-phase porous-cooling system. 1) $q = 3.5 \cdot 10^7 \text{ W/m}^2$, $\lambda_2 = 0.65 \text{ W/(m} \cdot \text{deg)}$; 2) $1.8 \cdot 10^7$ and 1.0 , respectively; 3) $8.3 \cdot 10^6$ and 2.6 .

Fig. 4. Thermal characteristic (dependence of the external heat flux on the position of the equilibrium phase-transition surface) of a two-phase porous-cooling system. 1) $q^* = 3.5 \cdot 10^7 \text{ W/m}^2$, $\lambda_2 = 0.65 \text{ W/(m} \cdot \text{deg)}$; 2) $1.8 \cdot 10^7$ and 1.0 , respectively; 3) $8.3 \cdot 10^6$ and 2.6 ; 4) from [17].

The hydrodynamic characteristics are plotted in normalized coordinates; the pressure drop is divided by $(P_0 - P_1)^* = 1.25 \text{ bars}$, which provides an equilibrium phase transition in the region with $l = 0.95$. The corresponding Reynolds number of the coolant flow is $Re = 0.1$. The coolant flow rates are divided by $G_1^* = 4.34 \text{ kg/(m}^2 \cdot \text{sec)}$, the flow rate of a liquid corresponding to a pressure drop of $(P_0 - P_1)^* = 1.25 \text{ bars}$ in viscous flow ($Re = 0.0$).

The sloping dashed curves ($l = \text{const}$) establish the correspondence between the coolant flow rate and the pressure drop at the plate for a fixed position of the phase-transition region. In particular, the curve $l = 1.0$ determines the drag of the plate for the single-phase liquid flow, while the curve $l = 0.0$ gives this drag for a flow of dry vapor. The gradual deviation of the dashed curves from their originally rectilinear shape reflects the increase of inertial drag as the flow regime changes from viscous (the Darcy regime) to transitional.

The hydrodynamic characteristics of all three systems intersect at the single working point a , because of the corresponding choice of the parameters λ_2 and q . For curves 1 and 2, however, the working point is on the ascending part, while for curve 3 it is on the descending part. According to the boundary conditions, constant pressure drop $P_0 - P_1 = 1.25 \text{ bars}$ is maintained at the plate. Then according to stability condition (2), curves 1 and 2 are stable at working point a , while curve 3 is unstable. For curve 2 there is yet another point where we have $P_0 - P_1 / (P_0 - P_1)^* = 1.0$ (point b), but it corresponds to an unstable regime. It should also be noted that curve 3 is unstable for coolant evaporation in any region within the plate, including the external surface ($l = 1.0$). Therefore, the method adopted in [14-16] for calculating the coolant flow rate from the heat-balance equation at the external surface is not appropriate for use in solving the thermal component of the process of two-phase porous cooling without a consideration of stability.

The conclusions drawn regarding the stability of a two-phase porous-cooling system on the basis of an analysis of its hydrodynamic characteristic agree fully with the results of an analysis of the intersection of the saturation curve and the temperature dependence of the pressure in the presumed phase-transition region. Furthermore, the hydrodynamic characteristic of the stable system can be used to find the permissible slow oscillations of the pressure drop along the plate. For curve 1, for example, the decrease in the delivery pressure is accompanied by a gradual deepening of the evaporation region, all the way to the state corresponding to the stability boundary (point d); then there is a rapid transition of the evaporation region to the internal plate surface, which causes a significant decrease in the coolant flow rate. Point d determines the minimum pressure drop for curve 1. Strictly speaking, within the framework of this model, only those permissible oscillations of the pressure drop which are due to oscillations of the delivery

pressure can be determined accurately. A pressure change of the external medium changes the physical properties of the coolant and thus the hydrodynamic characteristic. We can assume, however, that the error due to the permissible oscillations of the governing state is no larger than the error of the model itself; thus the permissible pressure fluctuations of the external medium can also be found, within the error with which the characteristic is plotted.

The solution of Eqs. (10) and (3) must be regarded as approximate, since important assumptions were made in the development of a model for the process. Nevertheless, these results are of considerable interest, since this problem not only cannot be solved in its exact formulation, it cannot even be formulated, because of a nearly complete lack of information on the behavior of the heat transfer and the resistance in the evaporation of a filtering coolant. The heat transfer and drag in the motion of even a single-phase coolant in a porous medium clearly require more study.

Thermal Characteristic of an Equilibrium Two-Phase Porous-Cooling System. The permissible fluctuations in the external heat load in a two-phase porous-cooling system can be determined from the static thermal characteristic of this system, i. e., from the heat flux as a function of the coordinate of the equilibrium phase-transition surface. Transforming (6), we find the basic equation for this calculation:

$$q = (i_l'' - c_m T_\infty) G \exp \left[\frac{G \delta c''}{\lambda_2} (1 - l) \right], \quad (12)$$

where the specific coolant flow rate G and the enthalpy i_l'' of the saturation vapor depend on the position of the phase-transition surface.

For a constant pressure drop along the plate the specific coolant flow rate changes markedly as the phase-transition surface moves. The corresponding equation is

$$G = - \frac{\mu'}{\beta/\alpha} \frac{m}{2n} \left(-1 + \sqrt{1 + 4 \operatorname{Re} \frac{n}{m^2}} \right), \quad (13)$$

where $\operatorname{Re} = [(P_0 - P_1)/(\delta \nu_3' \alpha)] [(\beta/\alpha)/(\mu')]$ is the Reynolds number of the coolant flow, and $m = [l + \nu''/\nu'(1-l)]$ and $n = [l + \rho'/\rho''(1-l)]$ are auxiliary complexes found by solving Eq. (3), which is of second degree in the specific flow rate G .

The change in the enthalpy of the saturated vapor as the evaporation zone moves within the porous wall is due to an increase in the saturation pressure. Equations for the increase in the saturation pressure and the corresponding change in the enthalpy of the vapor formed from the saturation parameters for evaporation at the external surface were obtained earlier: Eqs. (3) and (8) or (9), respectively. The linear approximation of the dependence of the enthalpy of the saturated vapor on the pressure gives us an analytic form of the equation for the desired static characteristic:

$$q = G \exp \left[\frac{G \delta c''}{\lambda_2} (1 - l) \right] \left\{ (i_{l=1}'' - c_m T_\infty) + \frac{di''}{dP} \Big|_{P_1} \delta \left[\alpha G \nu'' (1 - l) + \beta \frac{G^2}{\rho''} (1 - l) \right] \right\}. \quad (14)$$

If the coolant is water, the second term in braces vanishes, since the enthalpy of saturated water vapor remains essentially constant over a wide pressure range.

Figure 4 shows the thermal characteristics of those curves (1-3) whose hydrodynamic characteristics are plotted in Fig. 3 (1-3). For all these curves a constant pressure drop of $P_0 - P_1 = 1.25$ bars is maintained along the wall. The heat load for each curve is divided by the value q^* at which the phase-transition region is at $l = 0.95$. The corresponding normalizing values are given in the figure caption.

With $q = q^*$ the curves intersect at the common point a , but working point a lies on the descending parts of curves 1 and 2 and on the ascending part of curve 3. Furthermore, for curve 2 at $q = q^*$ a phase transition in the region with coordinate $l = 0.4$ (working point b) also satisfies the equilibrium condition. On the thermal characteristics the stability region is a descending region, as follows from the physical nature of the process. Thus a slight increase in the heat load in stable system 1 from steady state a causes the interface to move deeper; it returns to its initial position when the perturbations are subsequently removed. On unstable curve 3 the initial increase in the external heat flux involves a continuous advance of the evaporation zone toward the internal surface of the plate, since a lower heat supply to the external surface is required for an equilibrium phase transition in all intermediate states. The condition

for the stability of two-phase porous cooling in terms of the thermal characteristic is

$$\frac{dq}{dl} < 0. \quad (15)$$

This condition holds for a system with any type of coolant, but for a water-cooled system it is a simpler matter to plot the characteristic. An analogous condition for that particular case (without a generalization) has also been found by solving the linearized nonsteady-state system of equations, for a simpler formulation of the entire problem [17]. Curve 4 in Fig. 4 shows one result from [17]. The existence of yet another stability region for small values of l on curve 4 is due to the specification of a nonphysical boundary condition — the constancy of the temperature of the internal surface of the wall being penetrated.

It is a simple matter to find the limiting external heat load in a stable system from the thermal characteristic. A gradual increase in the heat load in [1] caused a gradual deepening of the evaporation zone, to $l = 0.6$ (point c); beyond this point even a slight perturbation in the external heat flux leads to boiling at the internal surface.

Study of the aperiodic stability of a two-phase porous-cooling system on the basis of various static characteristics leads to the same results. Each of these characteristics makes it possible to determine the permissible perturbation of one of the governing parameters in a stable system in a simple manner. With a set of characteristics available it is possible to find the permissible fluctuations in all the governing parameters and to make a multifaceted study of the process.

NOTATION

G	is the specific mass flow rate of coolant;
l	is the dimensionless coordinate of the phase-transition region;
δ	is the wall thickness;
P_0	is the delivery pressure;
P_1	is the ambient pressure;
q	is the external heat flux;
α and β	are the viscous and inertial drag coefficients of the porous structure;
Π	is the porosity of the wall material;
λ_2	is the effective thermal conductivity of the vapor region;
μ	is the dynamic viscosity;
ν	is the kinematic viscosity;
ρ	is the density;
c	is the specific heat;
r	is the total heat of vaporization;
i	is the enthalpy;
G_1	is the specific flow rate of the coolant in the Darcy regime;
Re	is the Reynolds number.

Subscripts and Superscripts

' and "	physical properties of the liquid and vapor, respectively, in the saturation state;
l	parameters in the phase-transition region;
*	normalized values.

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